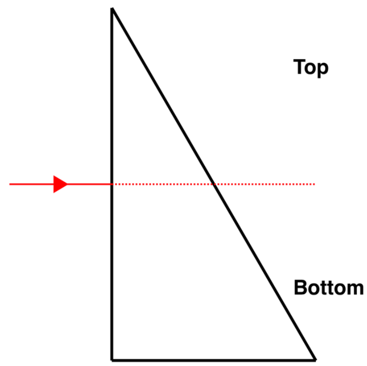
ReExam : Waves & Optics

18 April 2017, 18:30-21:30

- Put your name and student number on each answer sheet.
- Answer all questions short and to the point, but complete; write legible.
- Start each question on a new page.
- Answers that require a unit, but do not have one or the wrong one, are considered incorrect!
- All questions have equal weight.
- Final grade for this exam = 9 $\times$ total number of points/max number of points + 1

1. **Prism (18 points)**
Prisms were already used be the Romans to generate rainbows of color.



a) How are the refractive index, the local speed of light, the speed of light in vacuum, and the electric permitivity in vacuum and in a material related?
$n = v/c = \sqrt{\varepsilon_0/\varepsilon}\left(\sqrt{\mu_0/\mu}\right)$

b) In vacuum, how are the wave vector, electric field, and magnetic field oriented relative to each other?
$\vec{k} \perp \vec{E}$, $\vec{k} \perp \vec{B}$, $\vec{E} \perp \vec{B}$.

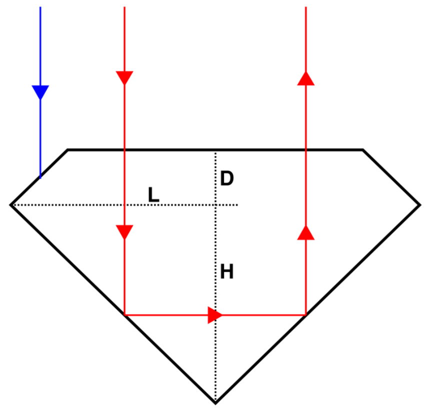
c) How is the irradiance of an traveling electromagnetic wave related to the instantaneous electric and magnetic field strenghts?
$I = \langle \vec{E}(t) \times \vec{B}(t) \rangle_T$.

d) State Snell's law of refraction. Please indicate what each parameter stands for.
$n_1 \sin\theta_1 = n_2 \sin\theta_2$. Here n stand for the refractive index, and θ the angle of incidence measured with respect to the normal at the interface.

e) If the refractive index of the prism is larger than that of its surroundings, will the ray of light bend towards the top or the bottom? Explain.
It will bend towards the bottom. This follows from Snell's law, which makes the angle of exit larger than the angle of incidence.

f) If $dn/d\omega < 0$, does red light bend more or less than blue light?
$\lambda\nu = c$ and $\omega = 2\pi\nu$. So when n goes down with larger ω, it goes up with larger λ. Red light has a larger wavelength than blue, so red will have a larger n, and this will bend more strongly.

2. **Glitters and Glamour (21 points)**
Diamonds are cut such that they produce an appealing reflection and refraction pattern. For an ideal cut $L = H$.



a) What is meant with "critical angle"?
The angle for which the angle of exit becomes 90°, which is for $\sin\theta_1 = \frac{n_2}{n_1}$. This is only possible when a ray of light leaves an optically dense medium to an optically thin medium, e.g. glass to air.

b) Consider the drawing above. For which refractive indices of the diamond will the central red ray undergo total internal reflection? Assume the diamond is suspended in air with $n_{air} = 1$.
The angle of incidence is 45°, which has to be larger than the critical angle. The critical angle is calculated from $\sin\theta_C = 1/n_{diamond}$. For a critical angle of 45°, the refractive index follows from $\sin 45° = 1/\sqrt{2} = 1/n_{diamond}$, so $n_{diamond} = \sqrt{2} \simeq 1.4$. Larger refractive indices lead to smaller critical angles. So for $45° > \theta_C = \arcsin(1/n_{diamond})$, it follows that $n_{diamond} > 1.4$ the diamond will exhibit TIR.

c) Diamond has a refractive index of 2.42. Determine whether the blue ray that enters the side facet on the top left exits on the top or the bottom of the diamond. Use $4D = H$, and that the ray enters the side-facet at an angle of 45°.
Use Snell's law to calculate angles. The internal angle is given by $n_{air}\sin 45° = n_{diamond}\sin\theta$, which yields $\theta = 17°$. If it hits the bottom, it will certainly exhibit TIR. It then hits the bottom right side, under an angle of 17°. Following time-reversal symmetry (rays may run in opposite direction), it follows immediately that no TIR will take place, but that the ray will exit under and angle of 45°. The ratio $4D = H$ makes no difference.

d) Explain whether the diamond will be equally "sparkly" when submerged in water ($n_{water} = 1.3$).
Less so. The condition for total internal reflection will change. The critical angle will change to $\sin\theta_C = 1.3/2.42$, so $\theta_C = 32°$.

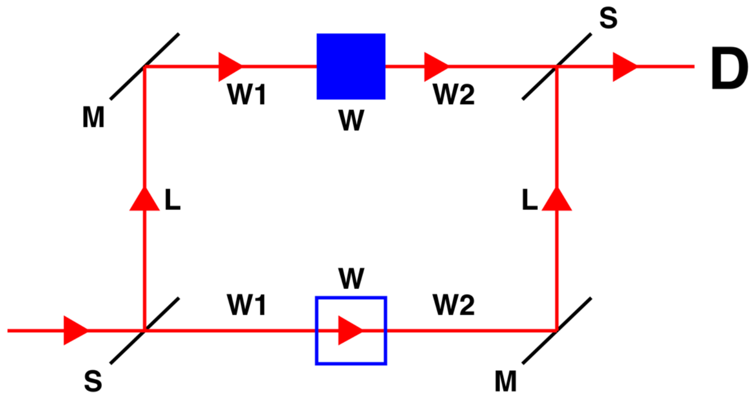
e) What is the difference between reflection coefficient and reflectance?
Reflectance $R = r^2$, with r the reflection coefficient.

f) What is meant with "Brewster angle"?
The angle at which no reflection occurs for the EM wave with its electric field in the plane of incidence. $\tan\theta_B = n_2/n_1$.

g) Does diamond reflect more or less light from its suface (at normal incidence) than glass, which has a refractive index of 1.6? Explain.

More. For normal incidence the reflection follows from Fresnels equantions: $R = |\frac{n_1 - n_2}{n_1 + n_2}|^2$. For diamond this is about 0.16, for glass 0.05.

3. **Interferometers (18 points)**
Interferometers are versatile devices that allow very sensitive measurements. In the figure below the schematic setup of a Mach-Zehnder interferometer is shown. The splitters labeled 'S' are 50% reflecting/transmitting, whereare those labeled 'M' are 100% reflecting. In the top leg a sample can be placed, indicated with the blue box. A similar, empty, box is placed in the bottom leg. At the top right a detector D is placed. Assume that the whole setup is placed in air.



a) What is meant with "constructive" and "destructive" interference (of two waves)? How does the irradiance of the resultant wave in both cases compare to those of the two separate waves?
Constructive: phase between two waves is about zero. Destructive: phase is about π. $A = A_1 + A_2 \cos \Delta\phi$, so $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \Delta\phi$, which follows from $I \propto A^2 = A_1^2 + A_2^2 + 2A_1 A_2 \cos \Delta\phi$.

b) How is "optical pathlength" defined?
$\Lambda = \int_{path} n(s) ds$.

c) If the sample box placed in the top leg is empty, what irradiance will the detector measure (relative to the irradiance at the entrance)?
It will be maximal, namely 50% of the entrance irradiance. The phase difference between the two legs is zero because the optical path lengths are the same.

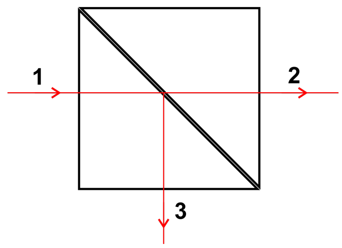
d) A transpartent substance with refractive index n_S is placed in the sample box. Derive an expression for the n_S-dependence of the irradiance observed at the detector.
The optical path length difference between the top and bottom ray is $\Delta\Lambda = Wn_S - Wn_{air} = W(n_S - 1)$. The corresponding phase difference is $\Delta\phi = 2\pi\Delta\Lambda/\lambda_0 = 2\pi(n-1)W/\lambda_0$. Here λ_0 is the wavelength in air. We then find that $I_D = I_{max}(1 + \cos \Delta\Phi)/2$.

e) For low density materials the refractive index can be approximated by

$$n^2(\omega) - 1 = \frac{Nq_e^2}{\varepsilon_0 m_e} \sum_j \frac{f_j}{\omega_{0j}^2 - \omega^2 + i\gamma\omega} \tag{1}$$

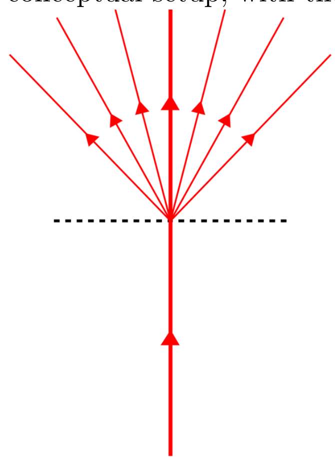
We can use this relation to measure the pressure of a gas placed in the sample cell. Which of the parameters is pressure-dependent? Explain. What do ω_{0j} and γ represent?
N is the electron density, which depends on the pressure. ω_{0j} stands for the resonance frequencies of the gas. γ represents the damping factor.

f) The beam splitters 'S' typically consist of two glass triangles, separated by a very thin layer of air. Explain the physical principle behind the splitting of the beam.



Total internal reflection + evanescent field.

4. **Diffraction (15 points)**
A transmission grating consists of bands of absorbing and transmitting material, each of width w. The sketch below shows the conceptual setup, with the incoming rays, and diffracted rays.



a) What defines a wavefront?
The plane in which the waves, *i.e.* oscillating electric and magnetic fields, have constant phase.

b) Give a (complete!) mathematical expression for the electric field strength of a traveling plane wave in 3D.
$\vec{E}(\vec{r}, t) = \vec{E}_0 e^{-i\omega t + \vec{k}\cdot\vec{r}}$. It is important to indicate vectors where necessary.

c) What are Fraunhofer and Fresnel diffraction?
Far-field and near-field diffraction, respectively. Near and far are defined in terms of the ratio between the wavelength and the size of the aperture.

d) Calclate w if the first diffraction maximum for a wavelength of 200μm is located at a diffraction angle of $45°$.
The transmitting stripes are $2w$ apart. The phase difference between them for an angle of 45 degrees must correspond to one wavelength. So $2w \sin 45° = \lambda$.

e) Explain why the width of the total grating determines whether you can separate two wavelenght that are close together, *e.g.* 200.0 and 200.2μm. Recall: $sinc(x) = \sin x/x$.
The narrower the grating, the wider the diffraction pattern for a fixed wavelength. The angular spread is given by $I(\theta) = I_0 sinc^2(x)$, with $x = kW/2 \sin\theta$. Here W is the total width of the grating and k the wavevector.